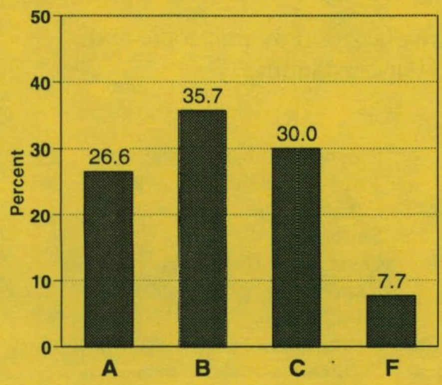
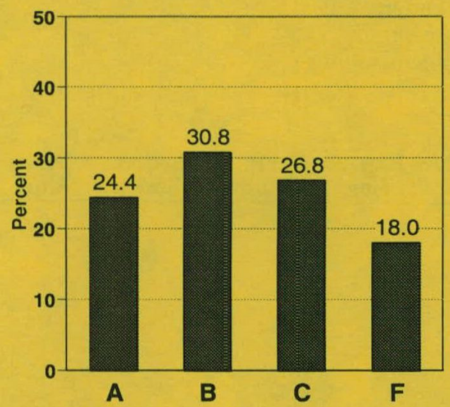
Mathematics 30

Diploma Examination Results
Examiners' Report for January 1998

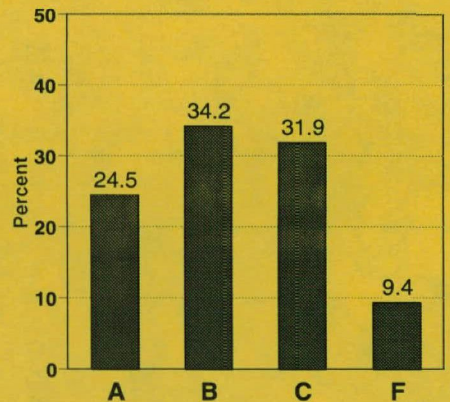
School-Awarded Mark



Diploma Examination Mark



Final Course Mark



The summary information in this report provides teachers, school administrators, and students with an overview of results from the January 1998 administration of the Mathematics 30 Diploma Examination. This information is most helpful when used with the detailed school and jurisdiction reports that have been provided electronically to schools and school jurisdiction offices. A provincial report containing a detailed analysis of the combined November, January, June, and August results is made available annually.

Description of the Examination

The Mathematics 30 Diploma Examination consists of 40 multiple-choice questions worth 57.1%, nine numerical-response questions worth 12.9%, and three written-response questions worth 30% of the total examination mark.

Achievement of Standards

The information reported is based on the final course marks achieved by 9 741 students who wrote the January 1998 examination.

- 90.6% of the 9 741 students achieved the acceptable standard (a final course mark of 50% or higher).
- 24.5% of the students achieved the standard of excellence (a final course mark of 80% or higher).

Approximately 49.9% of the students who wrote the January 1998 examination were female.

- 90.6% of the female students achieved the acceptable standard (a final course mark of 50% or higher).
- 24.3% of the female students achieved the standard of excellence (a final course mark of 80% or higher).

Approximately 50.1% of the students who wrote the January 1998 examination were male.

- 90.6% of the male students achieved the acceptable standard (a final course mark of 50% or higher).
- 24.7% of the male students achieved the standard of excellence (a final course mark of 80% or higher).

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Student Evaluation

Alberta
EDUCATION

Provincial Averages

- The average school-awarded mark was 69.0%.
- The average diploma examination mark was 66.0%.
- The average final course mark, representing an equal weighting of the school-awarded mark and the diploma examination mark, was 67.8%.

Of the 9 741 students who wrote the January 1998 examination, 11.56% had written at least one Math 30 exam previously.

Results and Examiners' Comments

This examination has a balance of question types and difficulties. It was designed so that students who are achieving the acceptable standard should obtain a mark of 50% or higher and students achieving the standard of excellence should obtain a mark of 80% or higher.

In the following table, diploma examination questions are classified by question type: multiple choice (MC), numerical response (NR), and written response (WR). The column labelled "Key" indicates the correct response for multiple-choice and numerical-response questions. For multiple-choice, numerical-response and written-response questions, the "Difficulty" indicates the proportion (out of 1) of students answering the question correctly.

Questions are classified by unit topic and mathematical understanding.

Unit Topic:

Poly. Fn.	Polynomial Functions
Trig. Fn.	Trigonometric and Circular Functions
Stat.	Statistics
Quad. Rltns.	Quadratics Relations
Exp. & Log.	Exponential and Logarithmic Functions
Perm. & Com.	Permutations and Combinations
Seq. & Series	Sequences and Series

Mathematical Understandings:

P	Procedure
C	Concept
PS	Problem-solving

Blueprint

Question	Key	Difficulty	Poly. Fn.	Trig. Fn.	Stat.	Quad. Rltns.	Exp. & Log.	Perm. & Com.	Seq. & Series	Math Und.
MC 1	B	0.721	✓							C
MC 2	D	0.888	✓							P
MC 3	A	0.785	✓							C
MC 4	B	0.616	✓							C
MC 5	B	0.383	✓							PS
MC 6	D	0.770	✓							P
MC 7	D	0.680	✓							C
NR 1	2143	0.621	✓							PS
MC 8	A	0.593		✓						P
MC 9	B	0.417		✓						PS
MC 10	D	0.622		✓						P
MC 11	D	0.798		✓						C
MC 12	C	0.608		✓						C
MC 13	C	0.563		✓						PS
NR 2	1.05	0.713		✓						P
NR 3	0.5	0.574		✓						C
MC 14	D	0.853					✓			C
MC 15	C	0.535					✓			PS
MC 16	A	0.926					✓			P
MC 17	D	0.802					✓			P
MC 18	C	0.706					✓			PS
MC 19	C	0.770					✓			P
MC 20	B	0.735					✓			PS
NR 4	1.7	0.532					✓			P
MC 21	D	0.690				✓				C
MC 22	A	0.780				✓				PS
MC 23	C	0.783				✓				PS
MC 24	A	0.709				✓				PS
MC 25	D	0.902				✓				C

Question	Key	Difficulty	Poly. Fn.	Trig. Fn.	Stat.	Quad. Rltns.	Exp. & Log.	Perm. & Com.	Seq. & Series	Math Und.
MC 26	C	0.468				✓				C
NR 5	1.5	0.723				✓				C
MC 27	C	0.739							✓	P
MC 28	C	0.813							✓	P
MC 29	B	0.639							✓	P
MC 30	A	0.666							✓	C
MC 31	A	0.464							✓	PS
MC 32	B	0.601							✓	PS
NR 6	350	0.789							✓	P
MC 33	B	0.711						✓		P
MC 34	A	0.800						✓		P
MC 35	A	0.873						✓		P
MC 36	C	0.253						✓		PS
MC 37	C	0.606						✓		P
MC 38	B	0.724			✓					C
MC 39	B	0.752			✓					PS
MC 40	A	0.580			✓					PS
NR 7	128	0.614						✓		P
NR 8	720	0.820						✓		C
NR 9	0.68	0.711			✓					P
WR 1	-									PCPS
WR 2	-									PCPS
WR 3	-									PCPS

Subtests: Machine Scored and Written Response (Average by Subtest)

When analyzing detailed results, bear in mind that subtest results **cannot** be directly compared. Results are in average raw scores.

Machine scored: 33.4 out of 49

Written response: 9.1 out of 15

Raw Score Average for Machine-Scored Questions by Course Emphasis

Poly. Fn	Polynomial Functions	5.5 out of 8
Trig. Fn	Trigonometric and Circular Functions	4.9 out of 8
Stat	Statistics	2.8 out of 4
Quad. Rltns	Quadratic Relations	5.1 out of 7
Exp. & Log.	Exponential and Logarithmic Functions	5.9 out of 8
Perm. & Com.	Permutations and Combinations	4.7 out of 7
Seq. & Series	Sequences and Series	4.7 out of 7

Raw Score Average for Machine-Scored Questions by Mathematical Understandings*

- Procedural (P): 13.9 out of 19
- Conceptual (C): 10.6 out of 15
- Problem Solving (PS): 8.9 out of 15

* Refer to Appendix C of the 1997–98 *Mathematics 30 Information Bulletin, Diploma Examinations Program,* for an explanation of mathematical understandings.

Multiple-Choice and Numerical-Response Questions

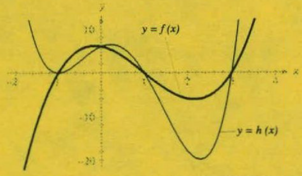
The following table gives results for four questions selected from the examination. The table shows the percentage of students in four groups that answered the question correctly. The comments following the table discuss some of the understandings and skills the students may have used to answer these questions.

Percentage of Students Correctly Answering Selected Machine-Scored Questions

	Question Number			
Student Group	MC 5	NR 1	MC 24	MC 26
All Students	38.3	62.1	70.9	46.8
Students achieving the *standard of excellence* (80% or higher, or A) on the whole examination	62.2	93.8	92.3	63.7
Students achieving the *acceptable standard* (between 50% and 79%, B or C) on the whole examination	32.3	61.6	68.2	44.0
Students who have not achieved the *acceptable standard* (49% or less, or F), on the whole examination	24.9	20.8	50.4	33.1

Use the following information to answer the next question.

The partial graphs of the polynomial functions $y = f(x)$ and $y = h(x)$ are shown below. All of the intercepts are integers.

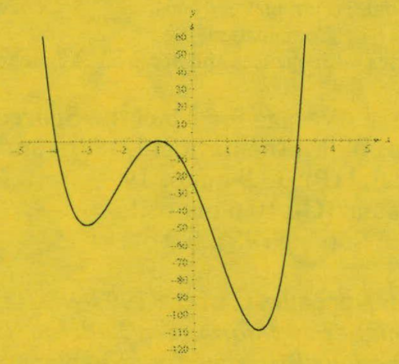


5. Which of the following expressions represents a relationship between $f(x)$ and $h(x)$?

A. $h(x) = -(x + 1)f(x)$
*B. $h(x) = (x + 1)f(x)$
C. $h(x) = -(x - 1)f(x)$
D. $h(x) = (x - 1)f(x)$

Use the following information to answer the next question.

The partial graph of a fourth-degree polynomial function, f, is shown below. The x-intercepts are –4, –1, and 3, and the y-intercept is –24.



Numerical Response

1. If $f(x) = a(x + b)^2(x + c)(x - d)$, where a, b, c, and d are all **positive**, then

the numerical value of a is ______ (Record in column 1 of numerical response 1)
the numerical value of b is ______ (Record in column 2 of numerical response 1)
the numerical value of c is ______ (Record in column 3 of numerical response 1)
the numerical value of d is ______ (Record in column 4 of numerical response 1)

(Record your answer as $\boxed{a}\boxed{b}\boxed{c}\boxed{d}$ in the numerical-response section of the answer sheet.)

Answer: 2143

The multiple-choice and numerical-response sections of the examination comprise questions that sample all content areas in Mathematics 30. A discussion of students' achievement of the curriculum standards in the units Polynomial Functions and Quadratic Relations follows.

Polynomial Functions — To achieve the acceptable standard in polynomial functions, students must be able to recognize and give examples of polynomial functions of different degrees; use the Remainder Theorem to evaluate a third-degree integral polynomial function for rational values of the variable and understand how this can be used to find factors of the polynomial function; factor and find zeros of an integral polynomial function in standard form, degree 3 or less, in which all zeros are rational; recognize the general shape of graphs of integral polynomial functions of degree 4 or less where the multiplicity of zeros is one, two, or three; and, determine the minimum degree of a polynomial function by using the multiplicities of its zeros. Multiple-choice questions 1, 2, 3, 5, 6, and 7 ask students to demonstrate their understanding of this unit.

In addition to the expectations for the acceptable standard, students who achieve the standard of excellence must be able to explain the relationships between the graphs of different polynomial functions and their zeros and recognize the general shape of graphs of integral polynomial functions of degree n where the multiplicity of zeros is greater than two. Multiple-choice question 4 and numerical-response question 1 ask this of students.

Quadratic Relations —To achieve the acceptable standard in quadratic relations, students must be able to describe orally, in writing, and by modeling, each of the following: the intersection of a plane and a conical surface that would result in a hyperbola, an ellipse, a parabola, and a circle. They must also be able to identify the position of the plane at which the intersection of a plane and a conical surface defines a degenerate ellipse and hyperbola. Students must be able to describe orally and in writing each of the following: the quadratic relation defined by a combination of numerical coefficients for any quadratic relation in the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where $B = 0$; state the quadratic relation formed when given the value of the eccentricity; state the eccentricity when given the quadratic relation; state the quadratic relation formed when given the locus definition; and state the effects on the graph of the quadratic relation in the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where $B = 0$, and TWO of the numerical coefficients change. Students are also expected to generate the graphs of quadratic relations with the use of graphing calculators or a graphing utility package; identify and graph the quadratic relation when given a point on the quadratic relation, a fixed point, and the eccentricity; calculate the eccentricity when given a fixed horizontal or vertical line, a fixed point, and a

24. Chris substituted four different values for C into the equation $x^2 + Cy^2 + Dx + Ey + F = 0$. The other coefficients were kept the same. Each new equation determined a non-degenerate conic.

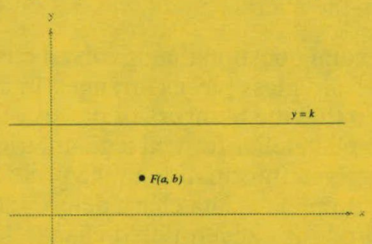
- The equation $x^2 + C_1y^2 + Dx + Ey + F = 0$ generated a hyperbola.
- The equation $x^2 + C_2y^2 + Dx + Ey + F = 0$ generated a parabola.
- The equation $x^2 + C_3y^2 + Dx + Ey + F = 0$ generated a circle.
- The equation $x^2 + C_4y^2 + Dx + Ey + F = 0$ generated an ellipse.

If C_1, C_2, C_3, C_4 form an arithmetic sequence, then the value of C_1 is

***A.** –1
B. –2
C. –3
D. –4

Use the following information to answer the next question

The graph of a parabola, $y = P(x)$, is used to model a relationship observed in an experiment. The directrix of the parabola is $y = k$, and the focus is $F(a, b)$, as shown below



26. The maximum value of $P(x)$ is

A. b
B. k
***C.** $\frac{k+b}{2}$
D. $\frac{k-b}{2}$

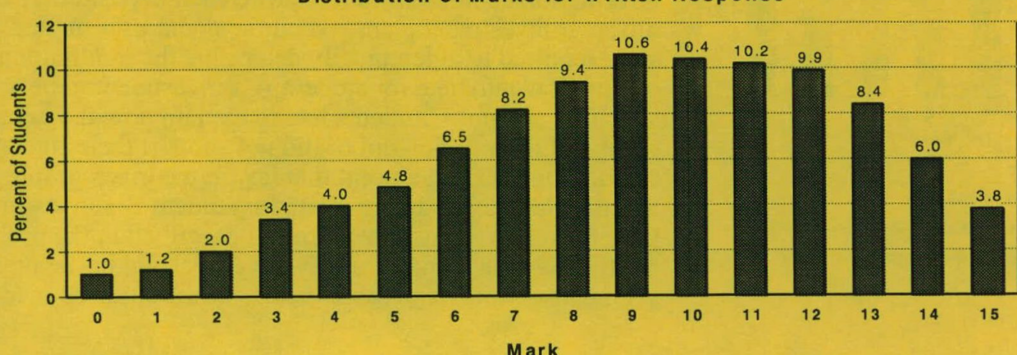
point on the quadratic relation; and identify and graph the quadratic relation when given the eccentricity, a fixed point, and a fixed horizontal or vertical line. Multiple-choice questions 21 to 25 and numerical-response question 5 required students to demonstrate their understanding of this unit.

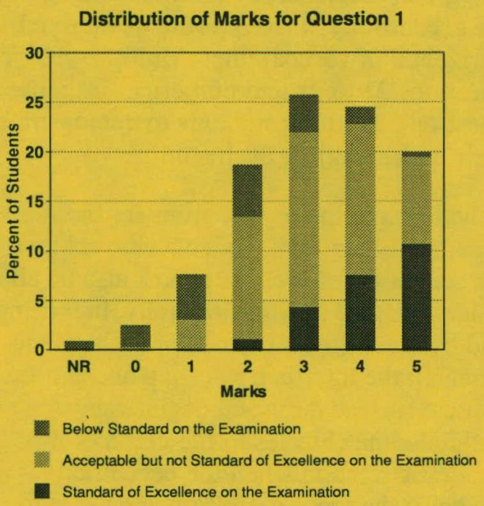
In addition to the expectations for the acceptable standard, students who achieve the standard of excellence must also be able to identify and to describe orally, in writing, and by modeling, the position of the plane at which the intersection of a plane and a conical surface defines a degenerate parabola; the changes in the graph of a quadratic relation when the eccentricity changes; the locus definition and use it to verify the equation of each conic section; the effects on the graph of the quadratic relation in the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where $B = 0$, when two or more of the numerical coefficients change; and, the solution to problems that require the analysis of quadratic relations studied in Mathematics 30. Multiple-choice question 26 required students to demonstrate these objectives.

Written-Response Questions

As published in the 1997–98 *Mathematics 30 Diploma Examination Information Bulletin*, the written-response questions assess whether or not students can draw on their mathematical experiences to solve problems and to explain mathematical concepts. Therefore, the written-response questions do not necessarily fall into a particular unit of study but may cross more than one unit or may require students to make connections among mathematical concepts. Students achieving the acceptable standard are expected to obtain at least half marks on all questions. Students achieving the standard of excellence are expected to obtain at least 12 out of 15 marks on the written response.

Distribution of Marks for Written Response

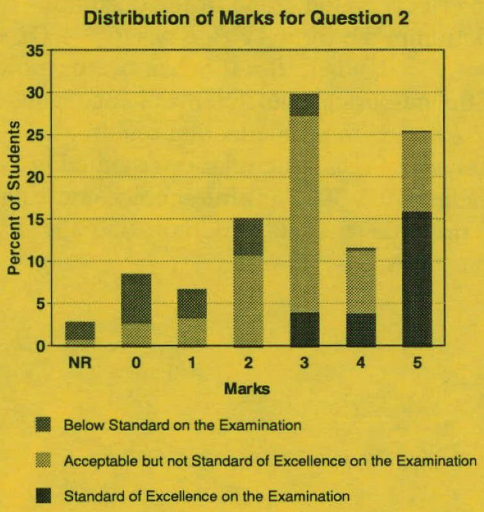




Question 1 required students to analyze two computer graphs, neither of which correctly represented the graph of a defined polynomial function, and identify which coefficients had been incorrectly entered. Justification of their reasoning was required. From a correct third graph of the same defined polynomial function, students were to estimate and list the x-intercepts, write the factored form of the polynomial, and algebraically verify that their factored form of the polynomial was correct.

It was expected that students achieving the acceptable standard would score 3 out of 5 marks. Of the students who met the acceptable standard of achievement on the examination, 78.7% received at least 3 out of 5 marks, and of students who achieved the standard of excellence on the examination, 76% scored 4 or more marks out of 5.

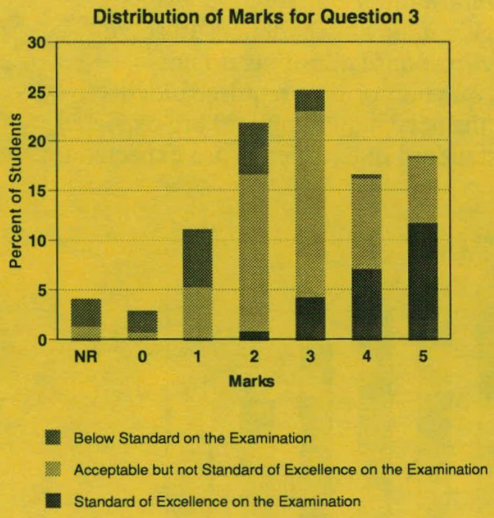
On this 5-mark question, the average mark was 3.2 or 64.1 %.



Question 2 was a contextual question that involved custom-made chandeliers made from glass pieces arranged in a circular formation in tiers. Given that the numbers of pieces of glass in each successive tier of a chandelier formed a geometric sequence, and using the given information in the problem, students were asked to complete a data chart, determine the cost of the glass pieces required for a seven-tiered chandelier, write a general expression for the cost of glass pieces required for a chandelier of n tiers, and apply statistical skills to determine the number of glass pieces that would be rejected by a manufacturing company, given the mean, standard deviation, and optimum width of the glass pieces manufactured. Some students made errors that resulted from not reading the problem carefully (and therefore using an arithmetic sequence) not using correct notation in the statistics portion, and not using the z-score table values.

Students achieving the acceptable standard were expected to score 3 or more marks out of 5, and students achieving the standard of excellence were expected to score 4 or more marks out of 5. Of the students who achieved the acceptable standard on the examination, 78.2% scored 3 or more marks out of 5, and of the students who achieved the standard of excellence on the examination, 82% scored 4 or more marks out of 5.

On this 5-mark question, the average mark was 2.9 or 59.9%.



Question 3 was also a contextual question that provided information about two students who each deposited $1 000 in different investment plans. With the given information, students were required to algebraically determine the minimum amount of time it would take for student A's investment to be worth a specified amount. Students had to explain how the given graphical information did or did not support their algebraic calculations. Given a point that lay on the given graph, students had to calculate the interest rate for student B's investment and by referring to the two investments described in the problem, explain what the lengths of given vertical and horizontal line segments on the graph represented. Some students made errors

by not reading the question carefully, and therefore not taking into account the specified domain in calculating the time for the investment to reach a minimum value; by misreading the given values of the ordered pair; by giving the ılculated percentage for the investment of student B as 1.03% or 0.03% instead of 3%; and by calculating the lengths í the line segments PQ and RS instead of explaining what these line segments represented.

Of the students who achieved the acceptable standard on the examination, 59.7% scored 3 or more marks out of 5, and of the students who achieved the standard of excellence on the examination, 77.8% scored 4 or more marks out of 5.

On this 5-mark question, the average mark was 2.8 or 57.6%.

Scoring Guide for Written-Response Questions

Credit may be given to students who show unusual insight. If their solutions fall outside *Specific Question Scoring Rubrics*, they were scored against the *General Scoring Guide* shown below.

GENERAL SCORING GUIDE

5 marks The student
- demonstrates a *complete understanding* of the problem
- uses mathematical knowledge and problem-solving techniques to find the solution
- justifies the solution and explain its relevance to the problem

4 marks The student
- demonstrates *an understanding* of the problem
- chooses a strategy that use mathematical knowledge and problem-solving techniques to find a solution, but the procedure contains *a minor flaw*
- shows *some justification* of his or her results

3 marks The student
- demonstrates *some understanding* of the problem
- formulates *some aspects* of the problem mathematically
- demonstrates the use of a strategy that uses mathematical knowledge and problem-solving techniques to find a *partial* solution
- communicates little understanding of the complexities of the problem

marks The student
- explores the *initial stages* of the problem
- applies *some* relevant mathematical knowledge and problem-solving techniques to find a *partial* solution

1 mark The student
- applies some relevant mathematical knowledge to the problem

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SPECIFIC QUESTION SCORING RUBRICS

Question 1

5 The student shows a complete solution by
- identifying for both graphs which coefficients were entered incorrectly and by explaining his or her reasoning
- listing the x-intercepts, writing $P(x)$ in factored form, and algebraically verifying the factored form to be equal to $P(x) = x^3 - 6x^2 + 3x + 10$

Note: Numerical substitution does not constitute an algebraic verification.

4 The student shows a solution by
- identifying for both graphs which coefficients were entered incorrectly, by explaining his or her reasoning, and by completing the third and fourth bullets, but the entire solution contains a minor flaw.

  Examples of minor flaws: – algebraic error in the verification
  – attempts all parts of the question but one part contains an error or omission, such as one explanation

3 The student shows a solution that contains a major flaw.
Examples of major flaws: – not answering bullet 4
– not answering bullet 1 or 2
– weak explanations in bullets 1 and 2

2 The student explores initial stages to find partial solutions by correctly completing
- bullet 4 (or bullets 3 and 4) **OR**
- two bullets of bullets 1, 2, and 3 (x-intercepts not sufficient)

1 The student explores only initial stages
- by correctly completing bullet 1 or 2 or 3 **OR**
- by writing the polynomial in factored form

Question 2

5 The student
- correctly completes the problems

4 The student finds a solution that contains a minor flaw such as:
- incorrect number in the chart **OR**
- an arithmetical error in the cost of a seven-tier chandelier **OR**
- rejecting $1 \times 100\,000(0.0228)$ pieces of glass

3 The student obtains a partial solution by
- correctly completing the first three bullets **OR**
- correctly completing any two of the first three bullets and by making significant progress on the fourth bullet **OR**
- correctly completing the fourth bullet and at least one of bullets 1, 2, or 3, or by making significant progress on two of bullets 1, 2, or 3 **OR**
- correctly completing the fourth bullet and consistently using a geometric sequence or arithmetic series all the way through the first three bullets

2 The student explores initial stages to find partial solutions by
- correctly completing one of bullets 2, 3, or 4 **OR**
- correctly completing the chart and by making significant progress on at least one other bullet **OR**
- making significant progress on two of bullets 2, 3, or 4 **OR**
- consistently and correctly using a geometric sequence or arithmetic series all the way through the first three bullets (4^{th} bullet not done) **OR**
- correctly completing the chart, using t_n in bullets 2 and 3, and making significant progress on the last bullet

1
- The student explores only initial stages of the problem; e.g., some makes progress on one of bullets 2, 3, or 4 **OR**
- The student correctly completes the chart.

Note: Significant progress in bullet 4 is going beyond the probability associated with the *z*-score.

Question 3

5 The student shows a complete solution by
- algebraically determining how long it will take for the money to be worth $2 100 and indicating how the graph supports the answer **AND**
- algebraically determining the interest rate is 3% **AND**
- giving a clear explanation of the significance of line segments $\overline{PQ}$ and $\overline{RS}$

4 The student finds a solution that contains a minor flaw such as
- not stating how the graph supports their answer **OR**
- not writting 3% **OR**
- not realizing $t \in N$ in first bullet **OR**
- making an computational error in either of the first two bullets

3 The student has a partial solution; for example, the student
- completes two bullets correctly **OR**
- completes one bullet correctly and the other two partially

2 The student explores initial stages to find partial solutions by
- completing one bullet correctly **OR**
- making significant progress on at least two of the bullets

1 The student explores only initial stages by
- applying some relevant mathematical knowledge to one of the bullets

Note: If a student doesn't indicate time as fixed for $\overline{PQ}$ **and** the value of investments being equal for $\overline{RS}$ in the explanation for bullet 3, the student has **not** answered the 3rd bullet.

For further information, contact Marion Florence (mflorence@edc.gov.ab.ca) or Corinne McCabe (cmccabe@edc.gov.ab.ca) at the Student Evaluation Branch at 427-0010. To call toll-free from outside of Edmonton, dial 310-0000.